Some Opinions on the Logic Structure of Mathematical Analysis

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Abstract: The In this paper, the logic structure of the mathematical analysis and its reason are considered. At first, from the essence of the world in sociology, explain the reason that the objects of this subject are functions according to the equivalence between the functions and determinate relationships. Then explain that the suitable and basic tool for studying functions is limit for it is a bridge between the macrocosm and microcosm. Furthermore, according to the three fundamental characters of relationship: stability, variation, and accumulation during a continuous change process, analyze the stability through functions' continuity with limit, describe the change of relationship via functions' derivatives, and argue the total effect of continuous change by integral. At last, discuss the other parts of the structure of mathematical analysis and give their causes, which show that the structure of mathematical analysis is a complete subject system with strong logic.

1. Introduction

Any subject has its own logic structure and the structure is usually derived by its own nature, so does mathematics. Today, more and more people admit that mathematics is an accurate language which can be used to describe the object more exact than any other nature language in quantity, and the language can also show the dynamic progress of anything considered. What's more, the language automatically obeys the science laws and mathematical logic[1-2].

Mathematical analysis, as one method of three basic important means, are used to describe the world in mathematical language, and has its owns subject, namely, functions to be studied. It is well known that function reflects some character of the relationship between two objects by quantity in number value for some purpose. Further, the function mirrors the whole dynamic process about some object for a certain goal. During the dynamic process, the situations of the relationship described by function may variate. How about the variation of dynamic process? The first problem is the span of the change. Is it small or large? It is about the stability of the relationship, that is the continuity of function. How to detail the the variation point to point in the overall process? This is the second problem for a dynamic process. Difference is not precise to show the change at some point or a very small span because it is just only a macro concept about change over a span of a process. The rate of transformation is more accurate to describe the variation of some relationship, because the variation rate can give not only the detail change at any point but also the difference over a span during the course. And the rate of change is derivative for a function. Sometimes, how to accumulate the total variation during the entire process? This is the third problem. The most effective methods is to accumulate all the quantity of variation in the whole process. The accumulation of variation for a relationship which expressed by a function is the integral of the function. Therefore, choose a right tool to study the stability, the rate of variation and the accumulation of variation about some relationship described by functions, which decided the logical structure of mathematical analysis[3].

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2. The Research Subject of Mathematical Analysis

From a philosophical point of view, anyone in the world can not be isolated, and must be a point in the net of relationship. Therefor, the nature of the world or the society is the connections among those who are in some certain range, where the range is decided by the goal in which people want to considered for. In general, the relationships among those objects are complex, they may be one to one, many to one, one to many and many to many. To any relationship between two objects, if the relationship is considered as a result following to another one for some goal, among these relationships, only the one-to-one and many-to-one types are decided. It's well known, if the relationship is one-to-many, considering one as the cause, the following results are many and the consequence is not decided, in the many-to-many relationships, which result does follow to the cause is also confused. So the one-one and many-to -one relationships are the decided and can be easily studied. In another words, the one-to-one or many-to-one relation is the basic foundation for all relationships which can be easily studied. This requires a suitable concept to be used to describe the relation when we talk about some problem for a decided goal in a certain range of objects. Function satisfies all of the above and derive for them.

According to the definition of function: let X and Y be certain sets, we say that there is a function defined on X with values in Y if , by virtue of some rule f, to each element $x \in X$ there corresponds an element $y \in Y$. In this case, the set X is called the domain of the function. The symbol x used to denoted a general element of the domain is called the independent variable. The element $y \in Y$ corresponding to a particular value of $x \in X$ is called the dependent variable, and it denoted $y = f(x)$. As the argument $x \in X$ varies, the value $y = f(x) \in Y$, in general, varies depending on the values of x[7]. So to analyze correctly the relationship between two object in quantity, if one amount is given, then the amount of the other object under this relationship (rule f) must be decided. So the definition of a function satisfies the requirement for description of a relationship between two objects for a certain goal, and by the rule f, the amount of the two objects for that target corresponding to the domain $D(f) = \{x | y = f(x), x \in X\}$ of variables set, the range $R(f) = \{y | y = f(x), x \in X\}$of function are decided. In another way, the function can not only reflect the relationship between two items in quantity according to the character and nature of the relationship, but also give expression to the dynamic process of the relationship for some target. It is right argument for study on a decided relationship for a certain purpose.

To the complex relationships, what people focus on the variation of the quantity of two objects for the goal according to the relationship. However, the relationship between two objects usually sustain in a span of a process, this need people to know not only the variation at one point of the process but also the variation of the whole process. All above these can be described by a mathematical concept, namely, function. Therefore, functions are the effective tools to express the relationship between some objectives for certain aim, and become the subject for mathematical analysis course. In another way, functions are the basic and suitable tools for arguing the basic relationship among the world[4–5].

3. The Research Tools for Functions

Since the research subject of mathematical analysis is function, and function is a kind of relationship between two objects for a certain goal in some sense, furthermore, the relationship lasts the whole process. So the tool is used to study function must contains the property that it can not only reflect the whole process in macro-scope view, but also detail at any point in micro perspective over the span of the proces[6]s.

When people consider a relationship between two objects in the quantity counted by number for some goal, so the whole process of relationship becomes a mapping from number to number, this is the form of function. In number to number mapping system, the any double matched points dot a point, in this sense, a function basing on a relationship is a graph consisted of points. From the viewpoint of mathematics, the points of a set can be classified in two types: accumulation point and isolated point. The isolated points reflect the relationship in discrete, namely, any point according to the certain relationship decided by function has no correlation to the other points. The accumulated point in fact is a limit point, which is defined that a point $x_0 \in \Gamma$ is called a limit point of graph Γ determined by a function for some certain goal if every open interval containing this point contains at least on point of graph Γ distinct from the point x_0. From the definition of limit point, the accumulated point can not only show the relationship decided by the function in discrete but also reflect the correlation among the points in any neighbourhood of the accumulated point in continuity. The effective tool is selected as limitation is just proposed based on this factual significance, which can describe the relationship not only in micro-scope and macro-scope view but also in discrete and continuity methods during the whole process of a function. In some sense, limit is the fundamental concept on which the whole mathematical analysis ultimately rest [7]. As it known that the continuity, derivative and integral all defined through limit in different aspects.

4. The Logicsstructure of the Content of the Mathematical Analysis

In general, the calculus or mathematical analysis content are consist of six parts: the real number theory and its completeness, limit of sequence and function, the continuity of function, the derivative of function, the integral of function and the approximation theory. And the basic content of mathematical are continuity of function, the derivative and integral.

In the limit section, the limit of sequence is firstly discussed, which reflects the approximation of points in discrete. Then the limit of function derivative from the limit of real sequence, and the similar to the right and left limit of function at some certain real number point. In more sense, the limit of function is from point of view of the continuity. Between the sequence limit in discrete and the function limit in continuity, the bridge is the Heine lemma, where the necessary and sufficient condition to the existence of $\lim_{x\to x_0} f(x)$ is the existence of $\lim_{n\to\infty} f(x_n)$ and it is equal to the limit $\lim_{x\to x_0} f(x)$ for any neighborhood $U^0(x_0;\delta)$ as $\lim_{n\to\infty} x_n = x_0$, $\forall\delta \geq 0$ .Since the limit is defined by distance of real number, the real number theory, especially, the completeness of real number system can not be omitted. To detail the real number theory, the integer, rational number the connection among them also are introduced.

In the continuity section, the continuity is defined by limit of function at one point or even in a whole interval. Also the discontinuity is defined by limit of function in two types, the first class of discontinuity point such as removable discontinuity and jump discontinuity, the second discontinuity means that there does not exist at least one of the right limit or the left limit. The continuity means the stability of function, the existence of limit at one point means the stability in the neighborhood of this point, the continuity in the interval shows the stability of the relationship expressed by the function during the whole course of the interval. Where the stability means the value dose not change so big that their difference can not be measured. Sometimes the first class of discontinuity is called relative continuum, although two situation of them are not continuous, the different can be still counted. Only the second discontinuity is regarded as real instability. By the stability of functions, we know some properties of the relationship which are described by functions such as intermediate value theorem, maximum and minimum theorem so on, which are useful in practical application.

In the derivative section, the derivative of function at a point is firstly defined by limit with the formula $f'(x_0) = \lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0}$, in fact, the derivative is a measure for the rate of change. In some sense, the change rate is more accurate to give expression to the variation of the functional relationship. At first, the change rate can show the quantity of variation with the span which the rate crosses over. The second, the rate of change can be used to express the variation at any point of the interval of the function, this means the rate of change is more effective than the difference when they are used to show the change of some relationship defined by the function. Furthermore, the derivative can also reflect the monotonicity, convexity and concavity. And the monotonicity can tell how and what the function change according to the function in certain rule, in another way, the monotonicity of function can predict the development of the relationship described by function. The convexity and concavity can show the tendency of the change about the function, and the tendency gives the variation about the monotonicity about function, it is also the basic theory for optimization theory. Through mean value theorem, the derivative is connected to the primitive function, and derivative bridge the derivative function to its original function [8], it is a powerful tool for computation in various application.

In the integral section, the integral is also defined by limit of the Riemann sum of function in some interval with the formula $\int_a^b f(x)dx = \lim_{\|T\|\to 0}\sum f(\zeta_i)\Delta x_i$, where $[a,b] = \bigcup_{i=1}^{\infty}[x_i, x_{i+1}]$ and $\Delta x_i = x_{i+1} - x_i$, $\|\mathrm{T}\| = \sup\{\Delta x_i, i = 1,2,\cdots\}$, the integral is a measure for the total effect of almost continuous change. In general, integral is a useful method to get the sum in quantity about a dynamic process. The integral part are composed of definite and indefinite integral, the indefinite integral is used to compute the definite integral with the Newton-Leibnitz theorem about integral, and the indefinite integral and the derivative are the foundation theorem of differential equation theory.

The foundation of calculus are the continuum, derivative and integral. And the stability, change and the sum are the main properties of function. Besides these properties, in the practical application, the function need to be approximated by some simple methods which can not only keep its properties but also be computed easily, especially by the methods it can be computed through computer in the fast speed and with the least memory capacity. This is required by the development of computer and real application. Therefore the series becomes one part of the mathematical analysis. According to linear algebra, in a linear space, if the basis is decided, any element of the space can be expressed by a linear combination of the basis. It is well known that $\{1, x, x^2, \cdots x^n, \cdots\}$ is the simplest linearly independent basis, in fact the computing speed of series $\sum_{i=0}^{\infty} a_i x^i$ is the fast, and the memory capacity for it is also the least. So power series is one of the kernel theorem of approximation. Sometimes, in reality, many phenomenon are explained by transmitting in the mode of wave, the sine and cosine function are the classical wave function, further, the sequence $\{1, \sin x, \cos x, \sin 2x, \cos 2x, \cdots \sin nx, \cos nx, \cdots\}$ is orthogonal and linearly independent, this hints that the sequence is a perfect basis for a linear space, and it is obvious that the set of continuous functions is a linear space.Therefore, the functions of description of wave phenomenon are expanded by Fourier Series, which satisfies the requirement of reality application. The power series and the Fourier Series are the two basic approximation theory. The approximation theory perfectly complete the calculus and it bridge the gap between theory and practice.

Besides one variable functions, multivariate functions are also the studied subject of mathematical analysis, the multivariate functions are more useful than one variable function in real life, and they are more effective to describe the real problems which contain more than one factor. As the similarity as one variable functions, the basic tool for study multivariate functions is limit, it is defined by accumulated points. To the limit in real number, the points accumulate to the limit only in two directs, but there are innumerable directions for the approximation to the limit point in the R^n space with $\mathrm{n} \geq 2$. So the limits are different according to their approximating directions, sometimes they have some common property among them but sometimes there is nothing among them, and one kind of limit is not the necessary condition for the other kind of limit. Therefore the multivariate limits are more complex than those of one variable's. Since the limits of multivariate are different, and the concepts of limit are also defined by in different ways. The derivatives and integrals of multivariate functions are also different from one variable function. Since the derivative is the rate of variation, the derivatives of multivariate functions are classified by the variables and directs into derivative of variables, namely partial derivatives, and directional derivatives through different limits of multivariate. The situations to integrals of multivariate functions are more complex. In a sense, integral is a kind of sum, so the integrals of multivariate function can be divided into line integral, surface integral and volume integral according to different forms of sum, and they are also divided into the first type integral and the second type integral according to whether the integrand and integral variable are vectors. Especially, the computing method for integral of multivariate functions are different from that of one variable, such as the Green Formula and Gauss Formula of multivariate integrals to the Newton- Leibniz Formula. To the approximation theory about multivariate functions, there is no effective method as that for one variable functions for there is not a suitable basis for multivariate functions, although there is also a Taylor's Formula for multivariate functions.

5. Conclusion

In a word, the mathematical analysis study the functions which reflect the relationships composed of one factor or several factor. With limits, the course study the stability of the relationship as function's continuity, the variation of the relationships with change rate as functions' derivatives and the sum of change as functions integral. It is well know that the world is a net of relationship, the simple decided relationship are one-variable function and multivariate functions, from this point of view, mathematical analysis is the most basic course of Higher Mathematics. And its logical structure develops from the nature properties of relationship.

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